WORK IT OUT

A maths workbook — concepts, worked examples, graded practice and number puzzles

Do the Learn First pages before the practice. Do the practice before the puzzles. Full solutions are at the back — but they only help you after you have tried.

Part	What it is	How to use it	Time
1	Learn First	Read and annotate. Copy every worked example by hand.	25 min
2	Worked Examples	Cover the solution. Try. Then compare line by line.	20 min
3	Guided Practice	Steps are given; you fill the gaps.	15 min
4	Practice Set A · Fluency	Straight application. Aim for speed and accuracy.	20 min
5	Practice Set B · Reasoning	Multi-step. Show every line.	25 min
6	Practice Set C · Challenge	Stretch problems. One may beat you — that is fine.	25 min
7	The Crossnumber	Answers are numbers. Wrong maths will not fit.	20 min
8	Number Search & Magic Square	Solo or pairs.	15 min
9	Spot the Mistake	Audit a worked solution. Find the planted error.	15 min
10	Quick-Fire Round	Matching, True/False, Two Truths and a Lie.	15 min
11	The Escape Challenge	Teams of 4.	25 min

Part 1 · Learn First

Two numbers, written in the right order inside brackets, can pin down any point in a flat plane exactly. This is the idea that let Brahmagupta's zero, Descartes' axes and a modern GPS receiver all speak the same language.

Two axes, one origin

A single number line locates a point on a line. To locate a point in a flat (2-D) space, we need **two** number lines at right angles to each other.

The **horizontal** line is called the **x-axis**; the **vertical** line is called the **y-axis**. The point where they cross is the **origin**, written O, with coordinates $(0, 0)$.

Distances from O are marked off in equal units on both axes. Distances **to the right** of O (or **upward** from O) are positive; distances **to the left** of O (or **downward** from O) are negative.

A point on the x-axis has the form $P(x, 0)$: if $x>0$ it lies right of O, if $x<0$ it lies left of O. A point on the y-axis has the form $P(0, y)$: if $y>0$ it lies above O, if $y<0$ it lies below O.

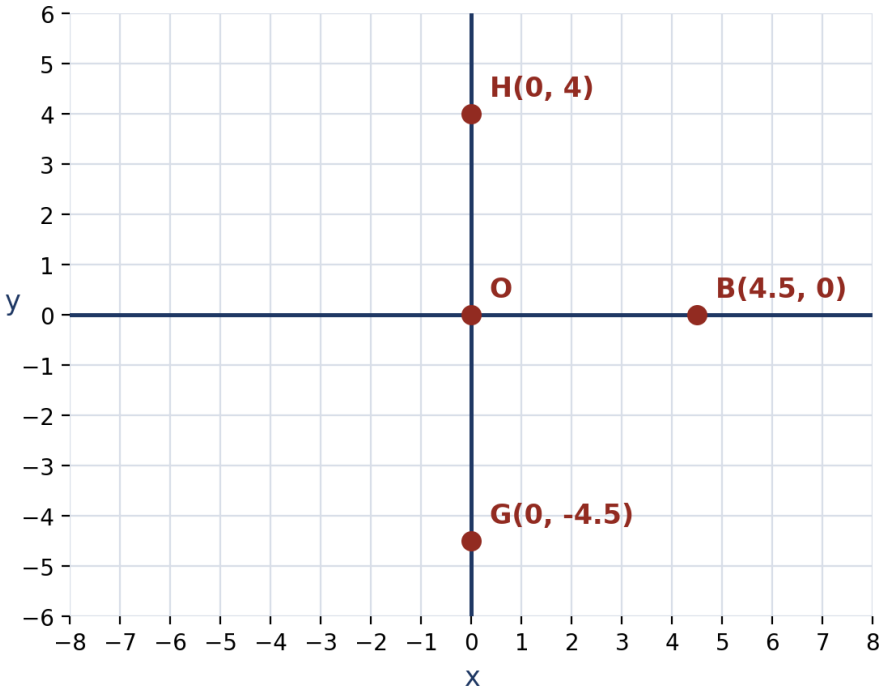

Fig. 1.1 — B(4.5, 0) on the x-axis; H(0, 4) and G(0, −4.5) on the y-axis; O is the origin

Rule to remember

- A point (x, y) is written with x first, then y — order matters.
- x is the **perpendicular distance from the y-axis**, measured along the x-axis.
- y is the **perpendicular distance from the x-axis**, measured along the y-axis.

- The origin O always has coordinates $(0, 0)$.

The four quadrants

The x-axis and y-axis together divide the plane — called the **Cartesian plane** or the **xy-plane** — into four regions called **quadrants**, numbered I to IV going anticlockwise from the top right.

A point can also lie **on an axis** rather than in any quadrant — its x-coordinate is 0 (on the y-axis) or its y-coordinate is 0 (on the x-axis). Such points are not counted in any quadrant.

Careful: (x, y) and (y, x) are the same point only when $x = y$. In general $(x, y) \neq (y, x)$ — swapping coordinates can move a point into a completely different quadrant.

Quadrant	x-coordinate	y-coordinate	Example
I	positive	positive	P(3, 4)
II	negative	positive	Q(−5, 3)
III	negative	negative	R(−4, −4)
IV	positive	negative	S(3, −5)

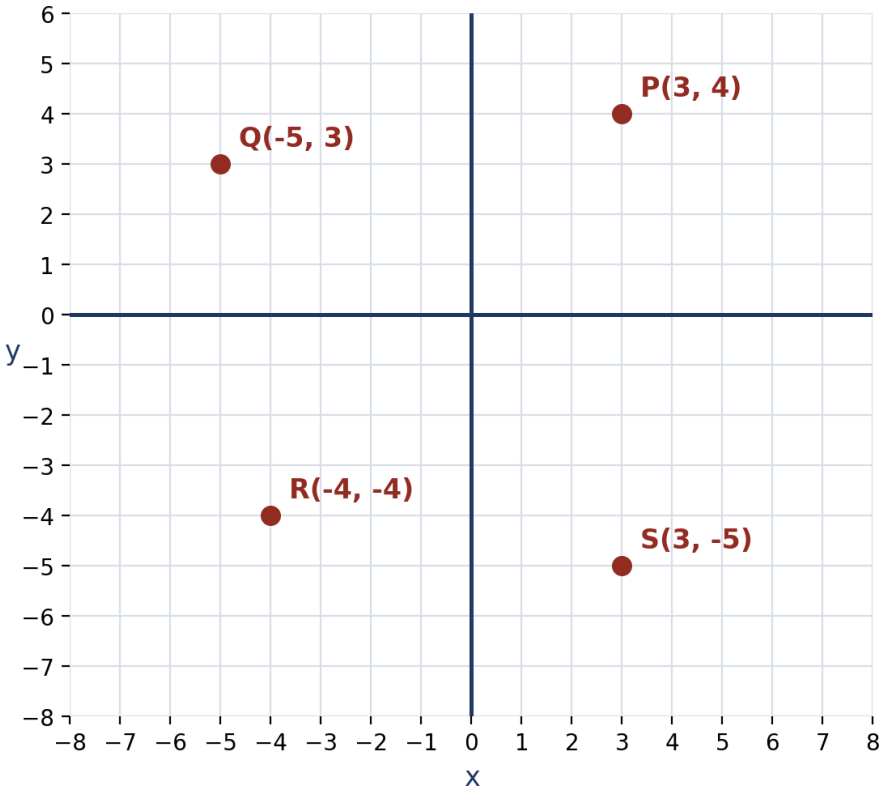

Fig. 1.2 — P(3, 4) in Quadrant I, Q(−5, 3) in Quadrant II, R(−4, −4) in Quadrant III, S(3, −5) in Quadrant IV

Sign convention, quadrant by quadrant

- Quadrant I: ¿
- Quadrant II: ¿
- Quadrant III: ¿
- Quadrant IV: ¿

Reading a point off a sketch

In Reiaan's room, door D1 is 8.5 ft from the y-axis (left wall) and 0 ft from the x-axis, since it lies on the floor line.

Its distance from the y-axis fixes the x-coordinate: $x = 8.5$.

Its distance from the x-axis fixes the y-coordinate: $y = 0$.

So D1 = $(8.5, 0)$ — a point on the x-axis, not inside any quadrant.

The habit to build: always read the x-coordinate from the y-axis, and the y-coordinate from the x-axis — it is easy to swap them by mistake.

Distance between two points

If two points share an x-coordinate or a y-coordinate, the distance between them is just a subtraction. If they do not, we fall back on the Baudhāyana–Pythagoras theorem.

For $A(3,4)$ and $D(7,1)$, drop a line from A parallel to the x-axis and one from D parallel to the y-axis; they meet at $C(3,1)$, forming a right angle at C.

Horizontal leg: $CD = 7-3 = 4$. Vertical leg: $AC = 4-1 = 3$. By the Baudhāyana–Pythagoras theorem, $AD = \sqrt{4^2+3^2} = \sqrt{25} = 5$ units.

In general, for points (x_1, y_1) and (x_2, y_2):

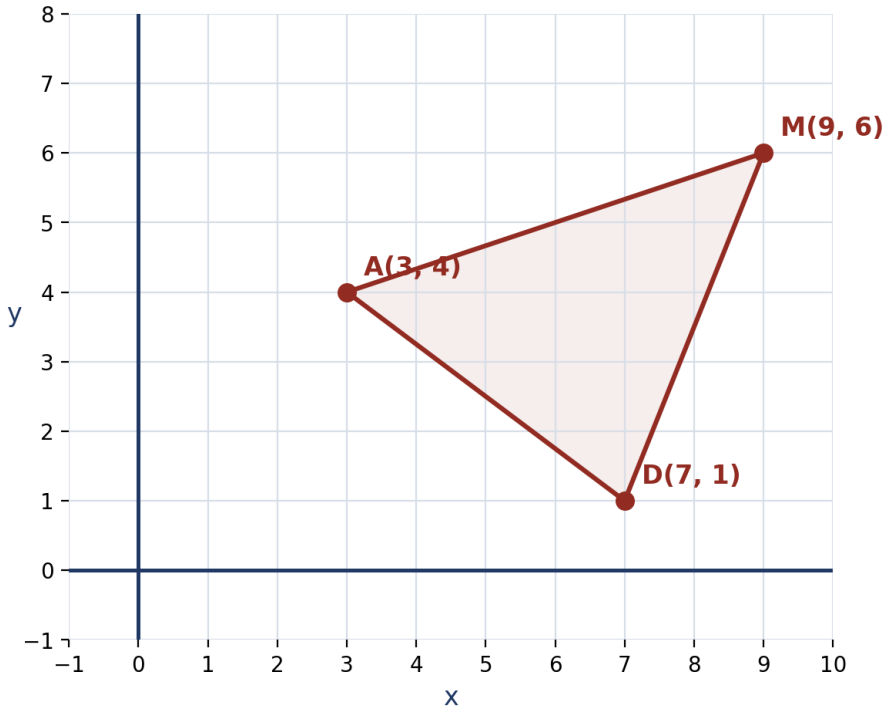

Fig. 1.3 — triangle ADM; the horizontal and vertical legs give each side by the Baudhāyana–Pythagoras theorem

The distance formula

- $Distance = \sqrt{\square\square}$
- It makes no difference whether $(x_2 - x_1)$ or $(y_2 - y_1)$ comes out negative — squaring removes the sign.
- If the two points share a y-coordinate, the distance is simply the difference of x-coordinates (and vice versa).

Distance between two points in different quadrants

Find the distance between $M(9,6)$ and $D(7,1)$.

Horizontal shift: $9-7=2$. Vertical shift: $6-1=5$.

$MD = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29}$ units.

$\sqrt{29}$ does not simplify further, so this is left as a surd.

Note: the formula works exactly the same way even when coordinates are negative — only the *difference* matters, not the individual signs.

Part 2 · Worked Examples

Cover the right-hand column with a sheet of paper. Try the problem yourself first. Then uncover it one line at a time and check where your reasoning parted from the model.

Example 1 — Plot the point $A(-4,5)$ and state which quadrant it lies in

Step 1. The x-coordinate is -4: move 4 units to the **left** of the origin along the x-axis.

Step 2. The y-coordinate is 5: from that spot, move 5 units **up**, parallel to the y-axis.

Step 3. Mark the point reached — that is $A(-4,5)$.

Step 4. x is negative, y is positive, so by the sign table this is Quadrant II.

Answer: $A(-4,5)$ lies in **Quadrant II**.

> **Why this works**
>
> Always move along the x-axis first, then parallel to the y-axis — mixing up the order is the single most common plotting mistake.

Example 2 — Find the coordinates of the point that is 6 units to the right of the y-axis and 3 units below the x-axis

Step 1. "6 units to the right of the y-axis" fixes the x-coordinate: $x=6$.

Step 2. "3 units below the x-axis" fixes the y-coordinate: since below the x-axis is negative, $y=-3$.

Step 3. Combine the two into an ordered pair, x first.

Answer: The point is $(6,-3)$, which lies in Quadrant IV.

> **Why this works**
>
> Reading these descriptions is exactly reading the x-axis distance and the y-axis distance separately, then writing them as (x,y) in that order.

Example 3 — Find the distance between $P(2,3)$ and $Q(5,7)$

Step 1. Horizontal shift: $x_2-x_1=5-2=3$.

Step 2. Vertical shift: $y_2-y_1=7-3=4$.

Step 3. Apply the distance formula: $PQ=\sqrt{3^2+4^2}=\sqrt{9+16}=\sqrt{25}$.

Step 4. $\sqrt{25}=5$.

Answer: $PQ=5$ units.

Why this works

This is the familiar 3-4-5 right triangle in disguise — recognising it saves you from leaving the answer as an unsimplified surd.

Part 3 · Guided Practice

The method is laid out for you. Fill in the blanks. If you can do these without looking back at Part 2, you are ready for Part 4.

1. Plot the point $B(3,-6)$ and name its quadrant.

x-coordinate is 3, so move _____ units to the _____ of O along the x-axis.

y-coordinate is -6, so from there move 6 units _____ , parallel to the y-axis.

x is _____ and y is _____ , so the point lies in Quadrant _____ .

---

2. State the coordinates of a point lying on the y-axis, 7 units below the origin.

A point on the y-axis always has x-coordinate _____ .

"7 units below the origin" means y = _____ .

The coordinates are _____ .

---

3. Find the distance between $A(0,0)$ and $B(6,8)$.

Horizontal shift: $6-0=¿$ _____ .

Vertical shift: $8-0=¿$ _____ .

$AB=\sqrt{¿¿^2+¿^2}=\sqrt{¿_¿}=¿¿$ _____ units.

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4. Determine the quadrant of the point $(-9,-2)$.

x-coordinate is _____ (positive/negative).

y-coordinate is _____ (positive/negative).

Both negative means Quadrant _____ .

Practice Set A · Fluency

Straight application of the method. Aim to get every one right, then aim to get them right quickly.

#	Question	Your working and answer
1	Write the coordinates of the origin.	
2	In which quadrant does the point $(-2, 7)$ lie?	
3	In which quadrant does the point $(5, -1)$ lie?	
4	What are the coordinates of a point that is 4 units to the left of the y-axis, on the x-axis itself?	
5	Plot the points $A(2, 2)$, $B(-2, 2)$, $C(-2, -2)$, $D(2, -2)$ and name the shape ABCD makes.	
6	Is $(3, 5)$ the same point as $(5, 3)$? Explain in one line.	
7	Find the distance between $(0, 0)$ and $(9, 12)$.	
8	Find the distance between $(1, 1)$ and $(4, 5)$.	
9	A point has x-coordinate 0 and y-coordinate -6. On which axis does it lie, and how far from the origin?	
10	Which quadrant contains all points with both coordinates negative?	

Practice Set B · Reasoning

Two or more steps, or a step you have to choose for yourself. Show every line — marks live in the working.

#	Question	Your working and answer
1	The three corners of a table are at $(8, 9)$, $(11, 9)$ and $(11, 7)$. If the fourth corner completes a rectangle, what are its coordinates?	
2	Find the distance between $M(9, 6)$ and $A(3, 4)$, leaving your answer as a surd if needed.	
3	A rectangular room has corners $O(0, 0)$, $F(18, 0)$, $R(18, 15)$ and $P(0, 15)$. What are its length and width?	
4	Points $B1(0, 1.5)$ and $B2(0, 4)$ mark the ends of a door. How wide is the door?	
5	Show that the points $(1, 1)$, $(4, 1)$ and $(4, 5)$ form a right triangle by finding all three side lengths.	
6	A point $P(x, y)$ satisfies $x<0$ and $y>0$. Which quadrant is it in, and what does this tell you about a point $Q(-x, -y)$?	
7	Two points $S(3, -5)$ and $S'(-3, -5)$ are reflections of each other. Across which axis, and how do you know?	
8	Find the distance between $D(7, 1)$ and $M(9, 6)$ from Fig. 1.3, and check it matches the value found for side DM in the notes.	

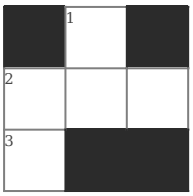
Practice Set C · Challenge

Harder, and deliberately so. If one defeats you, write down exactly where you got stuck. That sentence is worth more than a copied answer.

#	Question	Your working and answer
1	Triangle ADM has vertices $A(3,4)$, $D(7,1)$, $M(9,6)$. Find all three side lengths and state (with reasons, not just a protractor) whether the triangle looks isosceles.	
2	If $x \neq y$, explain fully why $(x, y) \neq (y, x)$, and describe exactly what transformation of the plane takes (x, y) to (y, x).	
3	A dining table 5 ft $\times 3$ ft is placed exactly at the centre of a dining room that runs from $(0,0)$ to $(18,15)$. Find the coordinates of all four feet of the table.	
4	Prove that the distance formula $\sqrt{\square^2 + \square^2}$ gives the same value regardless of which point is labelled 1 and which is labelled 2.	
5	A point moves so that it is always twice as far from the y-axis as from the x-axis. Give three coordinate pairs for points satisfying this, choosing points in three different quadrants.	

Part 7 · The Crossnumber

Every answer is a number. One digit per square. Because answers cross, a wrong calculation simply will not fit — this puzzle marks itself.



ACROSS

1. The quadrant number in which (-2, 7) lies *(1 digits)*

2. Square of the distance between (1,1) and (4,5) *(3 digits)*

3. Distance between P(2,3) and Q(5,7) *(1 digits)*

DOWN

1. Distance between (0,0) and (12, 16) *(2 digits)*

2. Distance between (0,0) and (9, 12) *(2 digits)*

Bonus questions that did not fit the grid — work them out anyway: Distance between (0,0) and (3, 4); Distance between (0,0) and (6, 8); Distance between (0,0) and (5, 12); The quadrant number in which (5, -1) lies; Distance between (0,0) and (8, 15); x-coordinate of a point 9 units right of the y-axis

Part 8 · Number Search & Magic Square

Work out each answer, then find it hidden in the digit grid — across, down, diagonally or backwards.

8	1	2	4	6	2	8	0	1	1	9	4
8	3	2	1	5	2	9	9	1	3	5	1
8	1	9	0	9	3	1	7	7	8	6	5
9	7	9	7	5	4	3	0	2	3	2	0
1	9	4	8	7	5	7	4	0	9	1	1
8	6	5	2	5	2	7	6	0	1	8	9
5	1	5	5	9	7	9	7	1	1	4	7
1	0	4	9	7	4	6	5	0	7	5	2
9	9	1	7	0	3	4	2	3	6	6	7
2	1	2	7	6	8	5	4	2	6	8	4
6	5	6	3	2	1	2	2	3	3	0	7
9	2	4	4	0	2	6	8	5	9	9	5

☐ 100	☐ 25	☐ 10	☐ 13	☐ 15
☐ 17	☐ 20	☐ 29	☐ 5	☐ 9

Magic Square — every row, column and diagonal adds to the same total.

8		
3	**5**	**7**
		2

Magic constant: 15. Fill the empty cells.

Part 9 · Spot the Mistake

Each solution below reaches a wrong answer, and exactly one step is to blame. Find the first wrong step and rewrite it correctly. Marking other people's work is the fastest way to stop making their mistakes.

❌ Spot the Mistake

Plotting the point $(-3, 5)$

Step 1. Move 3 units up along the y-axis first.

Step 2. Move 3 units left along the x-axis, then 5 units up parallel to the y-axis.

Step 3. Mark the point reached — done.

_____ The first wrong step is Step _____ . What should it say?

_____

❌ Spot the Mistake

Finding the quadrant of $(4, -9)$

Step 1. x-coordinate 4 is positive.

Step 2. y-coordinate -9 is positive because distance cannot be negative.

Step 3. So the point is in Quadrant I.

_____ The first wrong step is Step _____ . What should it say?

_____

❌ Spot the Mistake

Distance between $A(1, 2)$ **and** $B(4, 6)$

Step 1. Horizontal shift: $4 - 1 = 3$.

Step 2. Vertical shift: $6 - 2 = 4$.

Step 3. $AB = 3 + 4 = 7$ units.

_____ The first wrong step is Step _____ . What should it say?

_____

Part 10 · Quick-Fire Round

Match each point to its quadrant or axis

Draw a line from each item on the left to its match on the right.

#	Left	Right (shuffled)
1	$(3,4)$	Quadrant II
2	$(-5,3)$	Quadrant I
3	$(-4,-4)$	Quadrant III
4	$(3,-5)$	x-axis
5	$(0,6)$	Quadrant IV
6	$(7,0)$	y-axis
7	$(0,0)$	Origin
8	Distance from $(0,0)$ to $(3,4)$	5

True or False Sprint

Everyone stands. Teacher reads a statement. True → stay standing. False → sit down.

#	Statement	Your call
1	The origin has coordinates $(0,0)$.	☐ True ☐ False
2	In an ordered pair (x,y), the y-coordinate is written first.	☐ True ☐ False
3	A point with a negative x-coordinate and positive y-coordinate lies in Quadrant II.	☐ True ☐ False
4	A point on the x-axis always has y-coordinate 0.	☐ True ☐ False
5	A point on the y-axis always has x-coordinate 0.	☐ True ☐ False
6	$(2,5)$ and $(5,2)$ represent the same point.	☐ True ☐ False
7	Quadrant III contains points where both coordinates are negative.	☐ True ☐ False
8	The distance between two points can be negative.	☐ True ☐ False
9	The distance formula is $\sqrt{¿¿}$.	☐ True ☐ False
10	If two points have the same y-coordinate, the distance between them equals the difference of their x-coordinates.	☐ True ☐ False

#	Statement	Your call
11	The point $(0, -5)$ lies in Quadrant IV.	☐ True ☐ False
12	Descartes formalised the idea that any point in a plane can be described by two numbers.	☐ True ☐ False

Two Truths and a Lie

Two statements are true, one is false. Circle the lie — then correct it.

Set 1

(a) Quadrant I has both coordinates positive.

(b) Quadrant III has both coordinates negative.

(c) A point on the x-axis lies in Quadrant I or IV.

The lie is _____ . Corrected: _____

Set 2

(a) (x, y) and (y, x) are always different points.

(b) The origin is $(0, 0)$.

(c) Distances to the left of the origin are negative.

The lie is _____ . Corrected: _____

Set 3

(a) The distance formula comes from the Baudhāyana–Pythagoras theorem.

(b) Distance between two points can be found by simple subtraction only if they share an x-coordinate or a y-coordinate.

(c) Squaring a negative shift in the distance formula gives a negative result.

The lie is _____ . Corrected: _____

Part 11 · The Escape Challenge

Five gates, each a calculation. Each gives you one digit. Collect all five to unlock the exit code.

🔒 GATE 1 — The Origin Gate

What is the y-coordinate of the origin?

Digit: ☐

🔒 GATE 2 — The Quadrant Gate

In which quadrant (numbered 1-4) does the point $(-6, -8)$ lie?

Digit: ☐

🔒 GATE 3 — The Axis Gate

A point $(0, y)$ lies on which axis — give x-coordinate value that defines it.

Digit: ☐

🔒 GATE 4 — The Distance Gate

Find the distance between $(0, 0)$ and $(6, 8)$.

Digit: ☐

🔒 GATE 5 — The Shift Gate

For $A(2, 3)$ and $B(7, 3)$, what is the distance AB (they share a y-coordinate)?

Digit: ☐

EXIT CODE: ☐ ☐ ☐ ☐ ☐

Formula Sheet

Cut this page out and keep it. Everything in this chapter, on one page.

Origin

$$O=(0,0)$$

Point on x-axis

$$P(x,0)$$

Point on y-axis

$$P(0,y)$$

Distance formula

$$\sqrt{¿¿}$$

Quadrant I sign

$$¿$$

Quadrant II sign

$$¿$$

Quadrant III sign

$$¿$$

Quadrant IV sign

$$¿$$

Full Solutions

Not just answers — working. Compare your method with this one; if they differ but both reach the right answer, make sure you understand why yours also works.

Part 3 · Guided Practice

1. right; downward; positive, negative, IV · **2.** 0; -7; (0, -7) · **3.** 6; 8; AB = √(36+64) = √100 = 10 · **4.** negative; negative; III

Practice Set A

1. $(0, 0)$.

2. Quadrant II (x negative, y positive).

3. Quadrant IV (x positive, y negative).

4. $(-4, 0)$ — it is on the x-axis, so y = 0.

5. ABCD is a square of side 4 units, centred on the origin.

6. No — $(3, 5)$ has x = 3, y = 5, while $(5, 3)$ has x = 5, y = 3. They are the same point only if the two coordinates were equal.

7. $\sqrt{9^2+12^2}=\sqrt{81+144}=\sqrt{225}=15$ units.

8. $\sqrt{3^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$ units.

9. It lies on the y-axis, 6 units below the origin.

10. Quadrant III.

Practice Set B

1. $(8, 7)$ — it shares its x-coordinate with $(8, 9)$ and its y-coordinate with $(11, 7)$.

2. $\sqrt{6^2+2^2}=\sqrt{36+4}=\sqrt{40}=2\sqrt{10}$ units.

3. Length ¿18 units (along OF), width ¿15 units (along OP).

4. $4-1.5=2.5$ units wide.

5. Sides: $(1, 1)$-$(4, 1)$ = 3; $(4, 1)$-$(4, 5)$ = 4; $(1, 1)$-$(4, 5)$ = $\sqrt{9+16}=5$. Since $3^2+4^2=5^2$, it is a right triangle.

6. $P(x, y)$ with $x<0, y>0$ is Quadrant II. Then $Q(-x, -y)$ has $-x>0, -y<0$, so Q is in Quadrant IV — the reflection of P through the origin.

7. Reflection across the y-axis: the y-coordinates (-5) match and the x-coordinates are opposite in sign (3 and -3).

8. $\sqrt{2^2+5^2}=\sqrt{4+25}=\sqrt{29}$ units — matches the value found for DM in the notes.

Practice Set C

1. $AD=\sqrt{4^2+3^2}=5$; $DM=\sqrt{2^2+5^2}=\sqrt{29}$; $MA=\sqrt{6^2+2^2}=\sqrt{40}=2\sqrt{10}$. All three sides are different lengths, so the triangle is scalene, not isosceles.

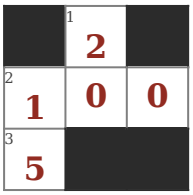
2. Swapping coordinates keeps the same two numbers but changes which axis each is measured along; unless $x = y$ this moves the point to a mirror position across the line $y = x$, a genuinely different point.

3. Room centre is $(9, 7.5)$. Table half-width ¿ 2.5, half-length ¿ 1.5, so feet are at $(6.5, 6), (11.5, 6), (11.5, 9), (6.5, 9)$.

4. ¿ and ¿ since squaring removes sign, so swapping which point is "1" and which is "2" leaves the sum, and hence the square root, unchanged.

5. For example $(4, 2)$ in Quadrant I, $(-6, 3)$ in Quadrant II, $(-8, -4)$ in Quadrant III — in each, |x| is twice |y|.

Part 7 · Crossnumber — Solved Grid



Across: 1 = 2 · 2 = 100 · 3 = 5

Down: 1 = 20 · 2 = 15

Part 8 · Number Search — Solved Grid

8	**1**	2	4	6	2	8	**0**	**1**	1	9	4
8	**3**	2	1	**5**	**2**	9	9	**1**	3	5	1
8	1	9	0	9	3	**1**	7	**7**	8	6	5
9	7	9	7	5	4	3	**0**	2	3	**2**	**0**
1	9	4	8	7	5	7	4	**0**	9	1	1
8	6	**5**	2	5	2	7	6	0	1	8	9
5	**1**	5	5	9	7	9	7	1	1	4	7
1	0	4	9	7	4	6	5	0	7	5	2
9	9	1	7	0	3	4	2	3	6	6	7
2	1	2	7	6	8	**5**	4	2	6	8	4
6	5	6	3	2	1	2	2	3	3	0	7
9	2	4	4	0	2	6	8	5	9	9	5

Part 8 · Magic Square — Solved (constant 15)

8	**1**	**6**

3	**5**	**7**
4	**9**	**2**

Part 9 · Spot the Mistake

Problem	First wrong step	It should say
Plotting the point $(-3, 5)$	Step 2	The x-coordinate is read along the x-axis, not the y-axis. Move **left along the x-axis first** (x is negative), then move up parallel to the y-axis.
Finding the quadrant of $(4, -9)$	Step 1	The y-**coordinate** can absolutely be negative — it just means the point lies below the x-axis. Here x is positive and y is negative, so the point is in **Quadrant IV**, not Quadrant I.
Distance between $A(1, 2)$ and $B(4, 6)$	Step 1	The distance formula uses the **Baudhāyana–Pythagoras theorem**, not plain addition: $AB = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ units, not 7.

Part 10 · Matching

#	Left	Correct match
1	$(3, 4)$	Quadrant I
2	$(-5, 3)$	Quadrant II
3	$(-4, -4)$	Quadrant III
4	$(3, -5)$	Quadrant IV
5	$(0, 6)$	y-axis
6	$(7, 0)$	x-axis
7	$(0, 0)$	Origin
8	Distance from $(0, 0)$ to $(3, 4)$	5

Part 10 · True or False

1-T · 2-F · 3-T · 4-T · 5-T · 6-F · 7-T · 8-F · 9-T · 10-T · 11-F · 12-T

Part 10 · Two Truths and a Lie

Set	The lie	Correction
1	c	A point on the x-axis (y = 0) does not lie in any quadrant — quadrants are open regions bounded by the axes, not including them.
2	a	(x, y) and (y, x) are the same point when $x = y$ — for example $(3, 3)$ maps to itself.
3	c	Squaring any real number, positive or negative, always gives a **non-negative** result — that is exactly why the formula works regardless of point order.

Part 11 · The Escape Challenge

EXIT CODE: 0 · 3 · 0 · 10 · 5

(Answer: 0) (Answer: 3) (Answer: 0) (Answer: 10) (Answer: 5)

You can now pin down any point in the plane with just two numbers, read off its quadrant on sight, and find the distance between any two points using the Baudhāyana–Pythagoras theorem. □